Simulation of Electromagnetic Cloaking using Transformation Optics

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ABSTRACT

Metamaterials are artificial material with a wide range of values of electromagnetic properties such as permittivity and permeability. They can be used to build unconventional electromagnetic devices such as electromagnetic cloaking. In this work, we study an electromagnetic cloaking system using a computational approach. To simulate electromagnetic cloaking, appropriate values of material electromagnetic properties must be used to manipulate electromagnetic wave to reduce the scattered field due to the cloaking system. We design an electromagnetic cloaking by using an optical transformation to obtain the required value of permittivity and permeability. Then we perform simulation of full-waveform electromagnetic cloaking using finite-difference time-domain (FDTD) method. The numerical result shows that there is still some scattered field due to the cloaking system. This problem is still needed to be solved in our future work.

INTRODUCTION

There are matematerials which can be achieved either vary large or vary small value of permittivity and permeability, including negative value. Metamaterials can also be obstructed to have continually various value of permittivity and permeability in spatial space. Because of these, metamaterial have a lot of applications. One interesting application of these materials is that they are possible to use in building electromagnetic cloaking device.

Invisibility or electromagnetic cloaking have been appealing from mankind for several years. There are many way to approach this system. Based on a coordinate transformation approach, Pendry et al. (2006) have reported and the simulation of Cummer et al. (2006) verifies this system. We study the system which use transformation optics to obtain parameter of device which metamaterials have been used in constructing as shown in Figure 1.

The outline of this paper is as follows. We first introduce metamaterials and transformation optics. Then numerical method used for simulation is presented followed by numerical results Finally a summary is drawn.

THEORY

Matematerials

Meatamaterial is a artificial structure whose dimensions are much smaller than the wavelength which interact with to satisfy the condition which we can treat the metamaterial as a average medium. This medium can be modeled by constitutive parameters. In case of electromagnetic, the medium is characterized by electric permittivity and magnetic permeability. The permittivity and permeability of material describe the response of medium to electromagnetic wave.

The common benefits of metameterials are that they are compound materials, we can construct material to have a various value of permittivity and and permeability, and the other is that they can provide inconvenient permittivity and and permeability.

When light travel through different materials or space it change direction. The parameters of material which involve the interaction between electromagnetic wave and that material are Electric permittivity and magnetic permeability. The permittivity and permeability of material could be derived from the methodology of transformation optic. This process will give us how those parameters of electromagnetic cloaking device cloud be.

Transformation optics

Consider a coordinate transformation from the Cartesian space (x, y, z) to an arbitrary curved space described by coordinates (u, v, w)

$$x = f_1(u, v, w),$$
$$y = f_2(u, v, w),$$

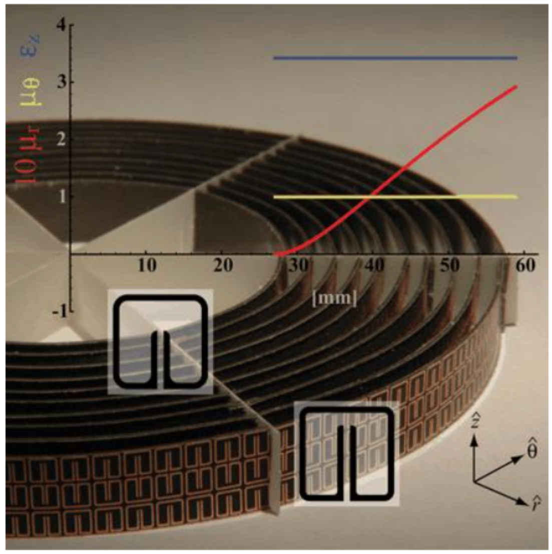

Figure 1: The experimental of transformation-based cloaking at microwave frequencies. (After Schurig et al. (2006))

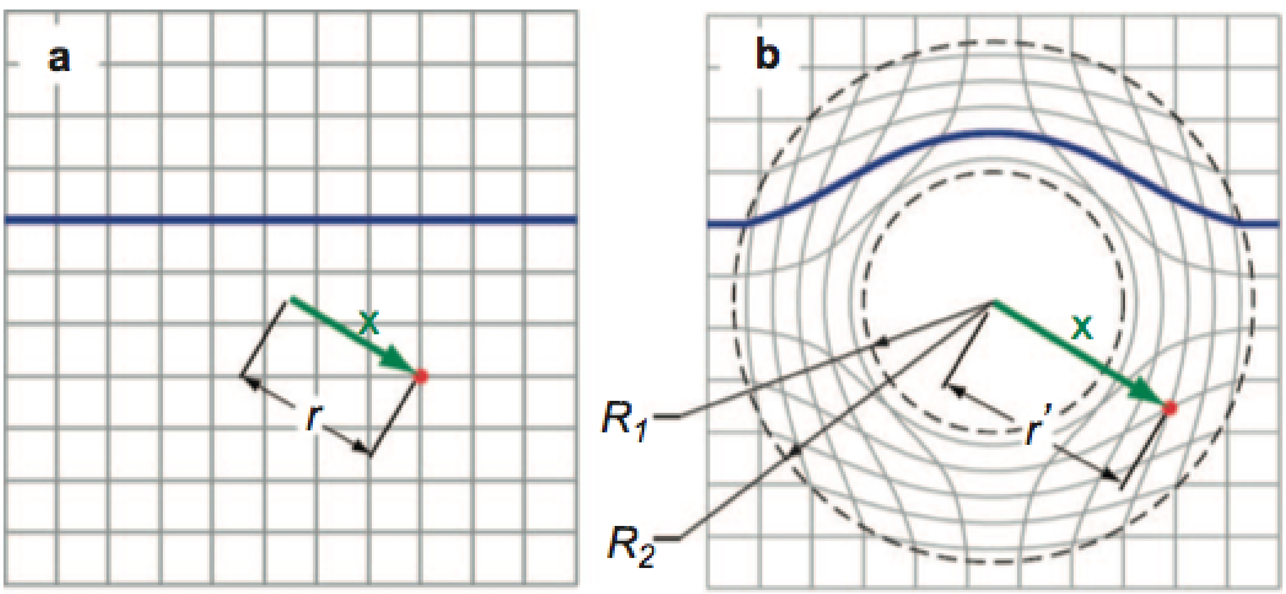

Figure 2: (left) Field line in free space with the back-ground Cartesian coordinate grid shown. (right) Distorted field line with the background coordinates distorted. (After Pendry et al. (2006))

$$z = f_3(u, v, w)$$

The Jacobian transformation matrix is written as

$$\Lambda = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{bmatrix} \tag{1}$$

In order to keep invariant forms of Maxwells equations, the new permittivity and permeability tensors have to be

$$\bar{\bar{\epsilon}}_{new} = \det(\Lambda)(\Lambda)^{-1}\bar{\bar{\epsilon}}(\Lambda)^{-T} \tag{2}$$

$$\bar{\bar{\mu}}_{new} = \det(\Lambda)(\Lambda)^{-1}\bar{\bar{\mu}}(\Lambda)^{-T} \tag{3}$$

We consider the comparison space as free space and then transform to real space.

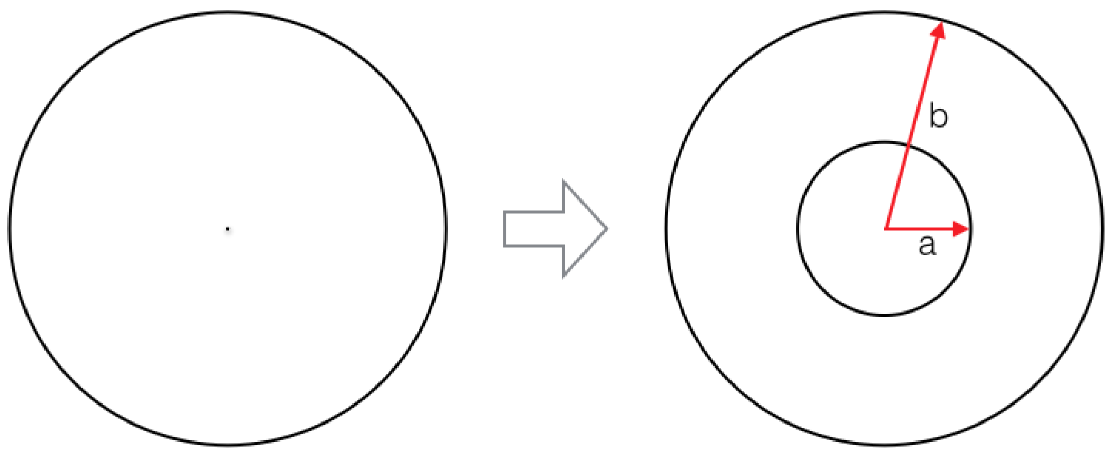

Figure 3: (left) comparison space, (right) real space.

The relation between two spaces should be (for linear transform)

$$r' = \frac{b-a}{b} r + a \tag{4}$$

The total Jacobian transformation could be

$$\Lambda = \Lambda_{xr} \Lambda_{rr'} \Lambda_{r'x'} \tag{5}$$

where Λ_{xr} denotes change of Cartesian coordinates to cylindrical coordinates, $\Lambda_{rr'}$ denote the transformation from the original cylindrical coordinates to new cylindrical coordinates, $\Lambda_{r'x'}$ denotes change of the new cylindrical coordinates back to Cartesian coordinates.

$$\begin{aligned}
\epsilon_{xx} &= \frac{r-a}{r}\cos^2\phi + \frac{r}{r-a}\sin^2\phi & (6)\\
\epsilon_{yy} &= \frac{r-a}{r}\sin^2\phi + \frac{r}{r-a}\cos^2\phi & (7)\\
\epsilon_{xy} &= \epsilon_{yx} = (\frac{r-a}{r} - \frac{r}{r-a})\cos\phi\sin\phi & (8)\\
\mu_{zz} &= \Big(\frac{b}{b-a}\Big)^2 \Big(\frac{r-a}{r}\Big) & (9)
\end{aligned}$$

another are zero.
note that: These are relative permittivity and relative permeability.

METHODS

We simulate electromagnetic wave using Maxwell's equation, Faraday's law and Ampere's law, as governing equations (eq 2.7). The other two equations describe interaction between electromagnetic wave and space.

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} \quad \text{(Faraday's law)} \tag{10}$$

$$\begin{aligned}
\frac{\partial \vec{D}}{\partial t} &= \nabla \times \vec{H} \quad \text{(Ampere's law)} & (11)\\
\vec{D} &= \bar{\bar{\epsilon}} \vec{E} & (12)\\
\vec{B} &= \bar{\bar{\mu}} \vec{H} & (13)
\end{aligned}$$

2D TE mode

For the 2D TE mode, we assume that there are no variations of either the fields or the excitation in z direction. Faraday's law and Ampere's law are reduced to

$$\begin{aligned}
\frac{\partial B_z}{\partial t} &= \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} & (14)\\
\frac{\partial E_x}{\partial t} &= \frac{\partial H_z}{\partial y} & (15)\\
\frac{\partial E_y}{\partial t} &= -\frac{\partial H_z}{\partial x} & (16)
\end{aligned}$$

Finite different time domain (FDTD)

FDTD was applied to solve the PDE

$$\begin{aligned}
\frac{\partial f}{\partial x} &\approx \frac{f(x+h) - f(x-h)}{2h}, & (17)\\
\frac{\partial f}{\partial t} &\approx \frac{f(t+\Delta t) - f(t)}{\Delta t}. & (18)
\end{aligned}$$

We proceed with discretization of Maxwells equations based on staggered grid as show in Figure 4.1.

the spatial positions of the two electric field components and the single magnetic field component. The E_x and D_x component is located at half x and integer y grid points, while the E_y and D_y component is located at integer x and half y grid points, and the magnetic component H_z and B_z is located at half x and half y grid points.

The spatially discretized of the component Maxwells Equa-

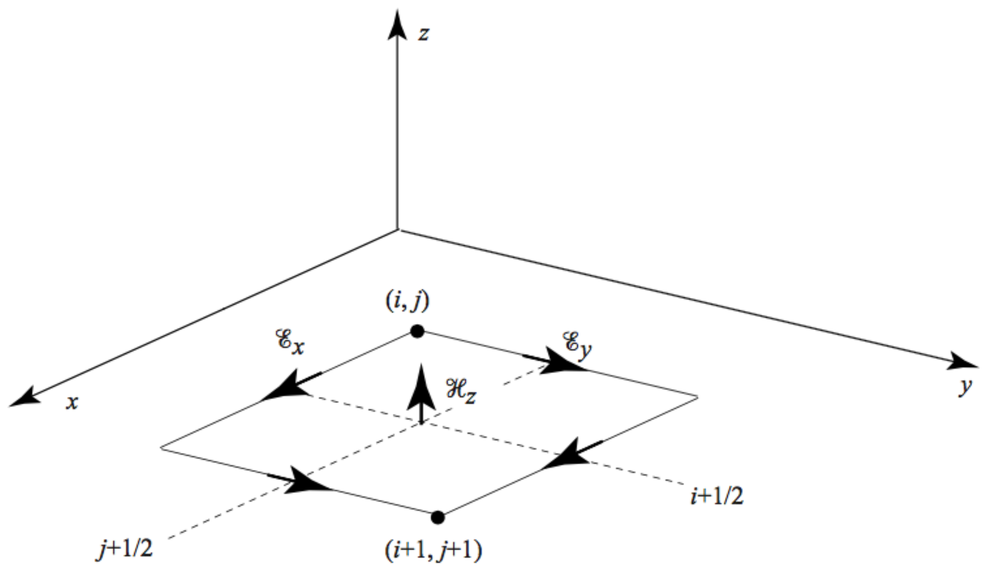

Figure 4: Staggered grid for an FDTD unit cell for transverse electric (TE) waves. (After Inan and Marshall (2011))

tions are

$$\begin{aligned} B_z^{n+1,i,j} &= B_z^{n,i,j} + \frac{\Delta t}{\Delta x}\left[E_x^{n,i,j+1} - E_x^{n,i,j}\right] \\ &\quad -\frac{\Delta t}{\Delta y}\left[E_y^{n,i+1,j} - E_y^{n,i,j}\right] \\ D_x^{n+1,i,j} &= D_x^{n,i,j} + \frac{\Delta t}{\Delta y}\left[H_z^{n,i,j} - H_z^{n,i,j-1}\right] \\ D_y^{n+1,i,j} &= D_y^{n,i,j} - \frac{\Delta t}{\Delta x}\left[H_z^{n,i,j} - H_z^{n,i-1,j}\right] \end{aligned}$$

Anisotropic media

The result form transformation optics provide an anisotropic media which is the parameters of media are function of vector direction of electric and magnetic field.
The permittivity and permeability are written in tensor form.

$$\bar{\bar{\epsilon}} = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \tag{19}$$

$$\bar{\bar{\mu}} = \mu_0 \begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \mu_{zx} & \mu_{zy} & \mu_{zz} \end{bmatrix} \tag{20}$$

From eq 2.7 we can written as

$$\frac{\partial \overline{D}}{\partial t} = \nabla \times \overline{H} \tag{21}$$
$$\overline{E} = [\bar{\bar{\epsilon}}]^{-1}\overline{D} \tag{22}$$
$$\frac{\partial \overline{B}}{\partial t} = -\nabla \times \overline{E} \tag{23}$$
$$\overline{H} = [\bar{\bar{\mu}}]^{-1}\overline{B} \tag{24}$$

Interface

Because we solve the problem in Cartesian coordinates, to get more accuracy about interface which is cylindrical we average the parameter at interface.

The result of this process is showed below (Figure 2.5).

NUMERICAL RESULTS

The cylindrical cloak was constructed in middle of square domain of the model. We generate plane wave form of electromagnetic wave at left of model. The right edge of model is absorbing boundary, one way wave. The top and bottom of model are Neumann's boundary to satisfy plane wave. Sitting of model is showed below.

The relative permittivity and relative permeability which we obtain from transformation are showed below (Figure 3.2). The parameter at inner boundary of a cylindrical cloak is inherently singular, make inverse value go to infinity. We avoid this problem by remove a thin layer from the inner boundary and replace with a thin perfect electric conductor (PEC) shell. The result of full wave form simulation are showed below. The result from our simulation are comparable to the result from simulation in Cummer et al. (2006) (Figure 3.4).

SUMMARY

Our simulate cloaking system without effect of dispersion of permittivity and permeability are comparable to the simulation in the simulation of Cummer et al. (2006). To predict and match experiment about the loss and the narrow bandwidth limit of metamaterials we have to implement dispersion effect of media.

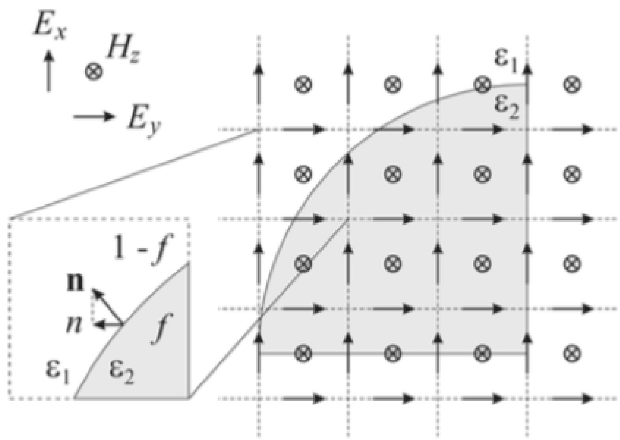

Figure 5: Parameter f is the ratio of material in each square area. (After Zhao and Hao (2007))

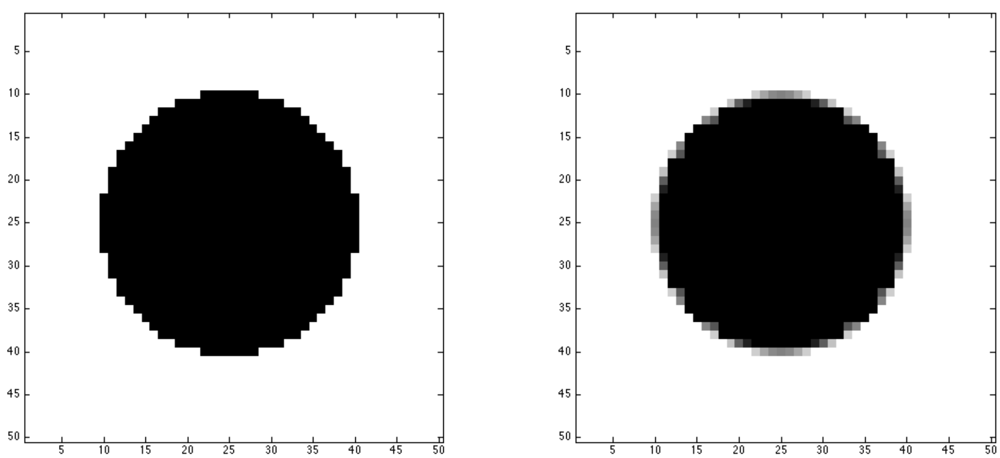

Figure 6: The result of average area, color show the ratio of material, black equal to one and white equal to zero

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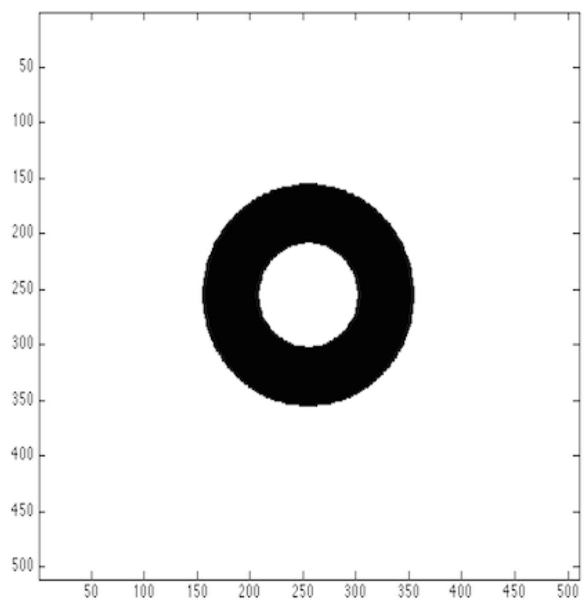

Figure 7: White is free space, Black is the cloaking device which have values a = 27.1 mm and b = 58.9 mm.

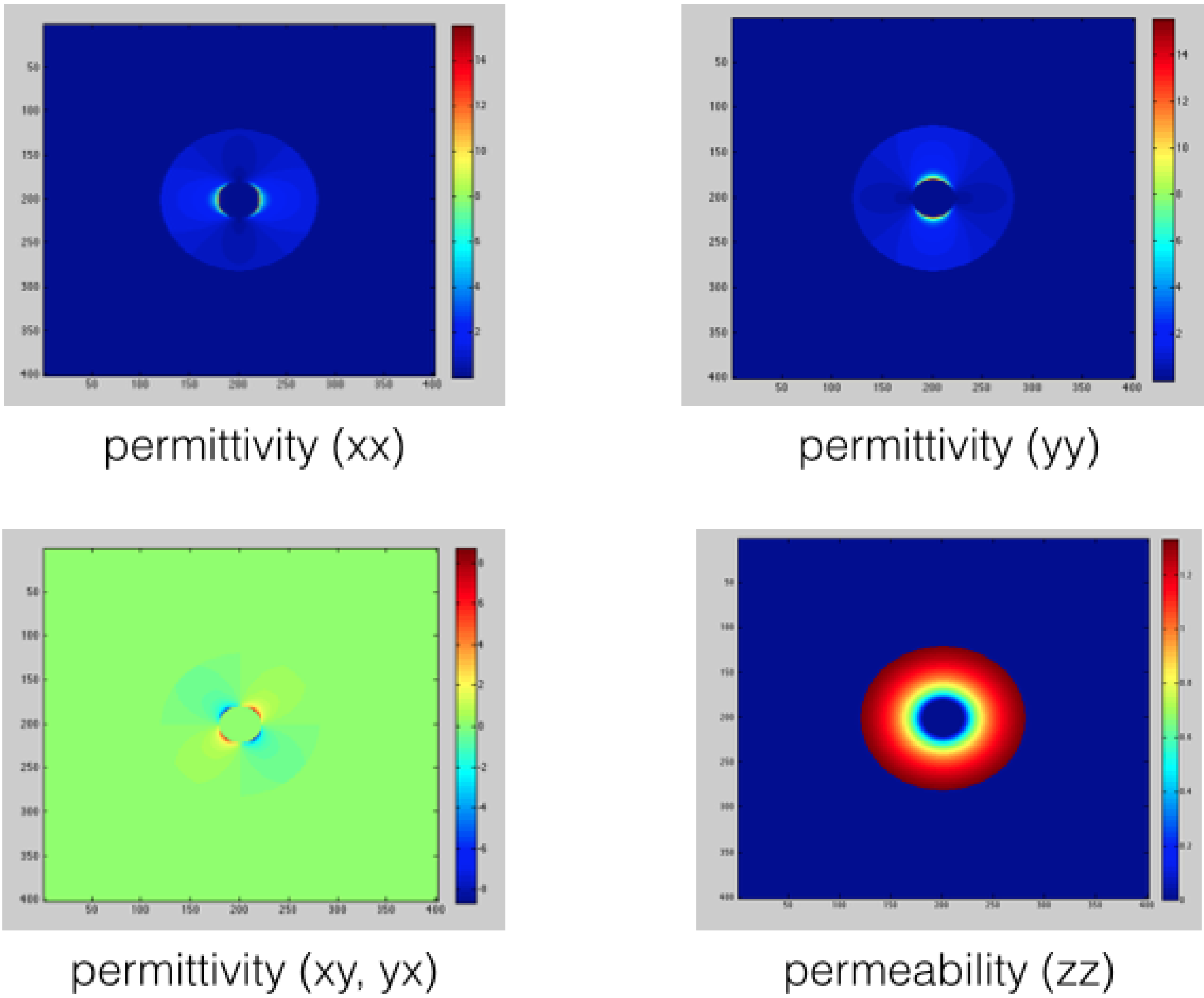

Figure 8: The relative permittivity and relative permeability in model

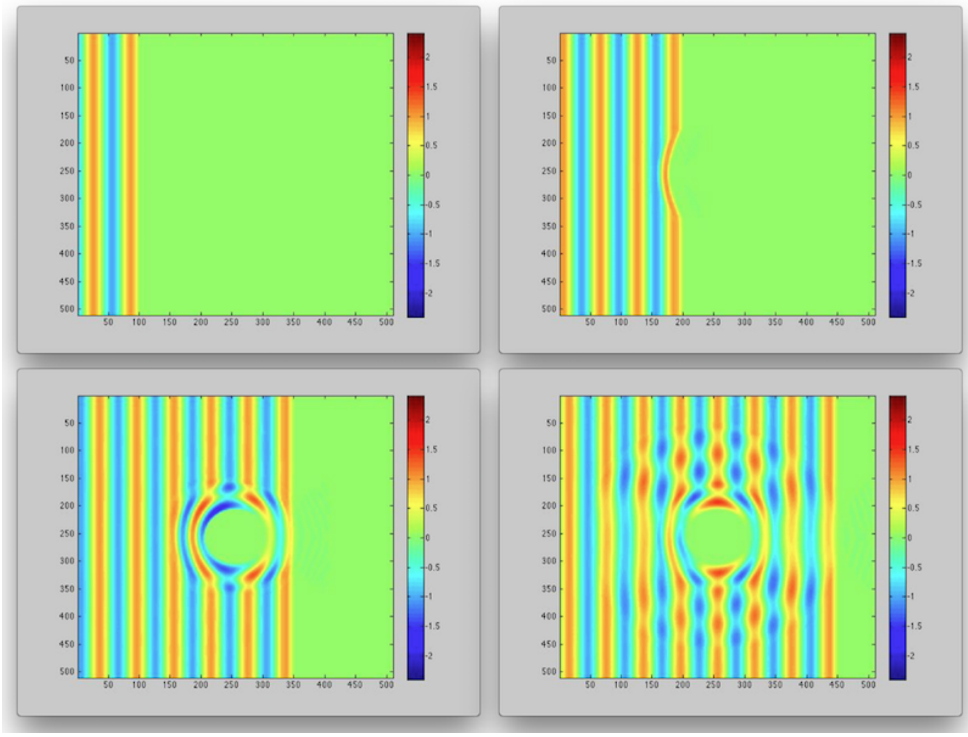

Figure 9: The simulation result with operating source 8.5 GHz of frequency. Source is plane wave that coming from the left of model.

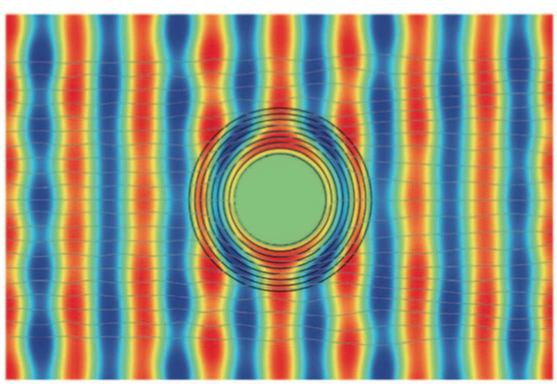

Figure 10: The simulation of approximation of the ideal parameters. (After Cummer et al. (2006))