Acoustic and Visco-acoustic Waveform Modeling in 2D Isotropic Media using the Finite-Difference Frequency-Domain (FDFD) Method

Chaiwoot Boonyasiriwat

ABSTRACT

Acoustic and visco-acoustic waveform modeling can be performed using the finite-difference frequency-domain (FDFD) method. The modeling process is efficiently performed implicitly by solving a linear system of equations using the LU decomposition when there are many source locations. Consequently, 2D full-waveform inversion (FWI) in the frequency domain is more efficient than its time-domain counterpart although this does not extend to the 3D case in which the computational of the direct solver is prohibitive. Nonetheless, frequency-domain modeling is still a very important tool for geophysical exploration and here I present a theory and application of frequency-domain modeling in two dimensions. The computer program developed in this work was also validated by numerical modeling of wave propagation in a synthetic model. Numerical results show that both phase distorsion and amplitude attenuation occur when the governing equation is the visco-acoustic wave equation.

INTRODUCTION

When seismic waves propagate in the earth, the wave amplitude decreases with the traveling distance due to the geometrical spreading and the anelastic property of rocks. The amplitude loss due to geometrical spreading can be easily compensated using a gain function. In contrast, the attenuation effect is more complicated than the geometrical spreading effect since attenuation is frequency dependent resulting to an accompanying of phase dispersion (Aki and Richards, 1980). High-frequency wave components will have higher energy loss compared to low-frequency components. In addition, the wavelet distorsion is more pronounced at longer time periods which correspond to the waves traveling from the deep regions. The problems caused by these amplitude decay and phase distorsion include the reduction in image resolution, and the mistie of well-derived synthetic seismograms. To overcome these problems, various inverse Q filtering methods have been proposed (e.g., Robinson (1979); Hargreaves and Calvert (1991); Wang (2002, 2006)). These inverse Q filters can be used for compensation of amplitude decay and/or phase distortion.

To apply inverse Q filtering to seismic data, an estimate of the quality factor Q is required. The simplest case is to use a constant Q model for attenuation compensation but the filtering result will certainly not be as good as when a more accurate Q model is used. A Q model can be estimated from vertical seismic profile (VSP) data (Wang, 2008), surface seismic profile (SSP) data (White, 1992; Dasgupta and Clark, 1994), or crosswell seismic profile data. Various methods for Q estimation have been proposed by many researchers including in-situ borehole measurements (Raikes and White, 1984; Stainsby and Worthington, 1985), the spectral ratio method (Wang, 2008), the frequency shift method (Quan and Harris, 1997), the amplitude-attenuation and amplitude-compensation methods (Wang, 2004). The in-situ borehole measurements provide the best estimate of the Q model but these data usually are not available. The spectral ratio method and the frequency shift method can be used in attenuation tomography to provide an estimate of the Q model in VSP, SSP, and crosswell cases. However, the estimated Q model usually has a low resolution due to the high-frequency approximation used.

Full-waveform inversion (FWI) can provide a high-resolution estimate of the Q model from seismic data especially the crosswell data (Wang, 2008). It is well known that, in two dimensions, full-waveform inversion in the frequency domain is more computationally efficient than the time-domain counterpart (Pratt, 1999). In addition, visco-acoustic waveform modeling can be easily performed in the frequency domain by replacing the real-valued velocity with a complex-valued velocity (Hicks and Pratt, 2001) while an auxilliary field is needed to perform visco-acoustic modeling in the time domain which leads to an extra computation time. Therefore, frequency-domain FWI is even more efficient than time-domain FWI for Q estimation in two dimensions. In three dimensions, the cost of the LU decomposition used to solve the large, sparse system of linear equations becomes prohibitive causing frequency-domain FWI based on direct solvers to be inefficient. One way to solve this problem is to use an iterative solver. An alternative to the frequency-domain approach is to perform the waveform modeling in the time domain.

In this paper, I present the theory and implementation of both acoustic and visco-acoustic waveform modeling using the finite-difference frequency-domain method in 2D isotropic media. The extension to 3D cases will be performed in the future.

ACOUSTIC WAVE EQUATION

In two dimenions, the constant-density acoustic wave equation can be written as

$$\frac{1}{v^2(x,z)}\frac{\partial^2 p(x,z|t)}{\partial t^2} = \frac{\partial^2 p(x,z|t)}{\partial x^2} + \frac{\partial^2 p(x,z|t)}{\partial z^2}, \tag{1}$$

where $p(x,z|t)$ is the pressure at position (x,z) at time t, and $v(x,z)$ is the wave velocity. To solve the wave equation in the frequency, we first apply the temporal Fourier transform to the wave equation 1 and obtain the so-called Helmholtz equation,

$$\frac{\omega^2}{v^2}P(x,z|\omega) + \frac{\partial^2 P(x,z|\omega)}{\partial x^2} + \frac{\partial^2 P(x,z|\omega)}{\partial z^2} = 0, \tag{2}$$

where $P(x,z|\omega)$ is the pressure in the frequency domain and ω is the angular frequency. Adding a source term $S(x,z|\omega)$ to the Helmholtz equation 2 yields

$$\frac{\omega^2}{v^2}P(x,z|\omega) + \frac{\partial^2 P(x,z|\omega)}{\partial x^2} + \frac{\partial^2 P(x,z|\omega)}{\partial z^2} = S(x,z|\omega), \tag{3}$$

ABSORBING BOUNDARY CONDITION

In this work, I used a simple absorbing boundary condition proposed by Engquist and Majda (1977),

$$\frac{\partial P}{\partial n} - ikP = 0, \tag{4}$$

where "n is the direction normal to the absorbing model boundary" (Ajo-Franklin, 2005), $i = \sqrt{-1}$, and $k = \omega/v$ is the wave number. This simple absorbing boundary condition is not very effective in reducing the spurious reflection from the domain boundary but it is very easy to implement and, therefore, I used it in this work. In the future, I will instead use the perfectly matched layer (PML).

FINITE-DIFFERENCE FREQUENCY-DOMAIN METHOD

To model the acoustic wave propagation in a visco-acoustic medium, the Helmholtz equation 3 is numerically solved using the finite-difference method. The spatial derivative $\frac{\partial^2 P}{\partial x^2}$ is approximated using the centered finite-difference scheme, given by

$$\frac{\partial^2 P}{\partial x^2} \approx \frac{P_{i-1,j} - 2P_{i,j} + P_{i+1,j}}{\Delta x^2}, \tag{5}$$

where P_{ij} is the pressure at $x = i\Delta x, i = 0,1,...,N_x$, and $z = j\Delta z, j = 0,1,...,N_z$, and Δx and Δz are the grid spacings in the x- and z-directions, respectively. In this work, we assume that $\Delta x = \Delta z$. Similarly, the spatial derivative $\frac{\partial^2 P}{\partial x^2}$ is approximated to

$$\frac{\partial^2 P}{\partial z^2} \approx \frac{P_{i,j-1} - 2P_{i,j} + P_{i,j+1}}{\Delta z^2}. \tag{6}$$

Inserting equations 5 and 6 into the Helmholtz equation 3 yields

$$\frac{\omega^2}{v^2}P_{i,j} + \left(\frac{P_{i-1,j} - 2P_{i,j} + P_{i+1,j}}{\Delta x^2}\right) + \left(\frac{P_{i,j-1} - 2P_{i,j} + P_{i,j+1}}{\Delta z^2}\right) = -S_{i,j}. \tag{7}$$

Equation 7 can be used to form a linear system of equation, written as

$$\mathbf{AP} = \mathbf{S}, \tag{8}$$

where $\mathbf{A}$ is called the stiffness matrix, $\mathbf{P}$ is the unknown pressure vector, and $\mathbf{S}$ is the source vector. LU decomposition was used in this work to solve the linear system 8 for the pressure vector $\mathbf{P}$.

NUMERICAL RESUTS

In this section, I applied the visco-acoustic modeling code developed in this work to model acoustic wave propagation in the homogeneous visco-acoustic medium with the quality factor of $Q = 500, 100, 50, 10$, and the velocity of 1000 m/s. The monochromatic source has the frequency of 20 Hz. The numerical results are shown in Figure 1.

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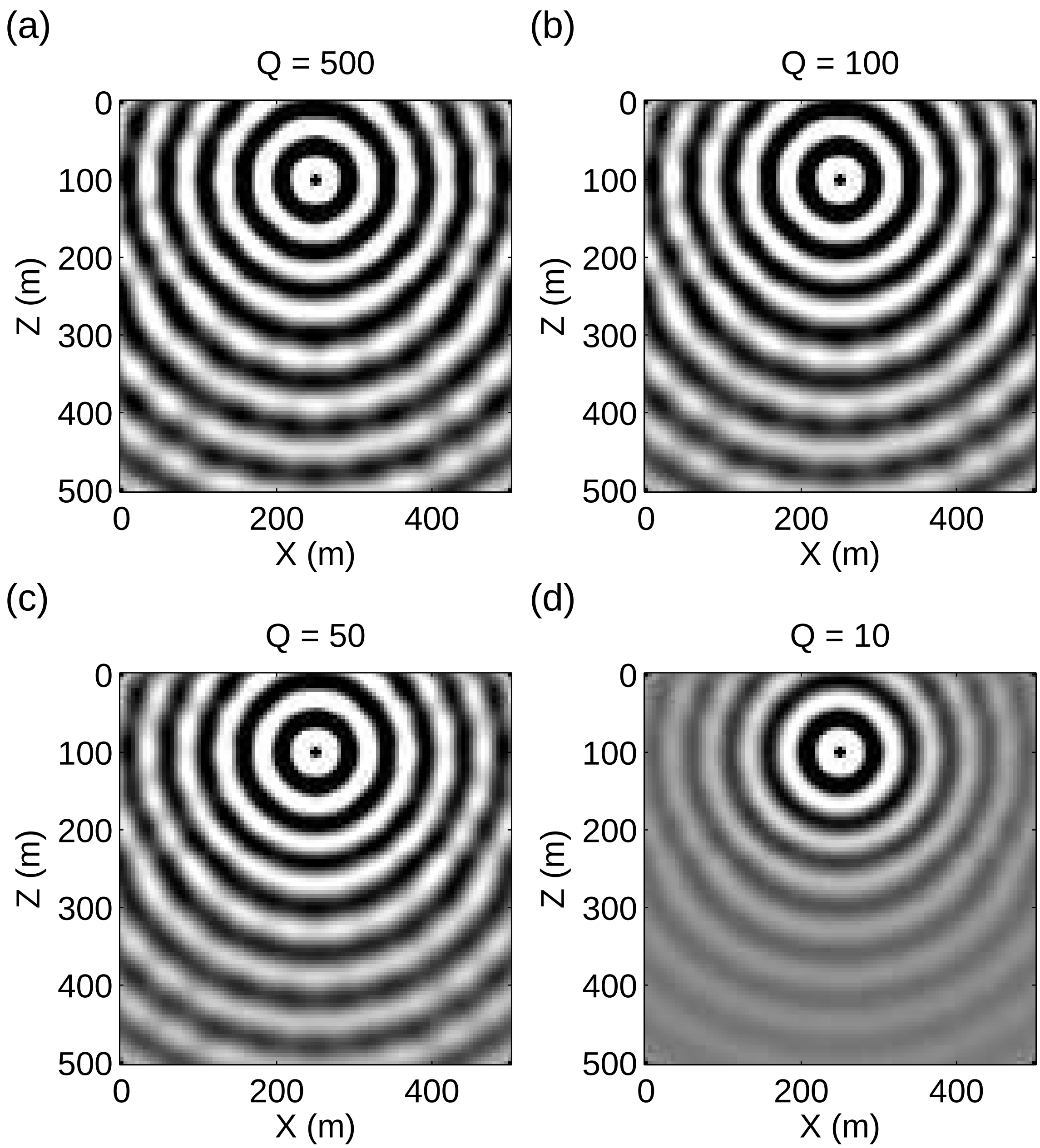

Figure 1: Acoustic wavefields in the homogeneous visco-acoustic medium with the quality factor of $Q = 500, 100, 50, 10$.

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APPENDIX A: DERIVATION OF COMPLEX PHASE VELOCITY FOR VISCO-ACOUSTIC MODELING

In this section, I briefly describe seismic attenuation models which are used in this work.

The anelastic attenuation factor or seismic quality factor denoted as Q quantifies the effect of anelastic attenuation on the seismic wavelet caused by fluid movement and grain boundary friction Sheriff and Geldart (1995). It is defined as

$$Q = 2\pi \frac{E}{\Delta E}, \tag{9}$$

where $E/\Delta E$ is the fraction of energy lost per cycle. Seismic attnuation is a dispersive behavior because the rate of attenuation increases with frequency.

In addition to the quality factor Q, the attenuation coefficient α is another commonly used measure of attenuation Toksoz and Johnston (1981). The quality factor Q and attenuation coefficient α are related as follows.

$$\frac{1}{Q} = \frac{\alpha v}{\pi f}, \tag{10}$$

where v is the velocity and f is the frequency.

The attenuation coefficient α is defined as the exponential decay constant of the amplitude of a plane wave traveling in a homogeneous medium. For plane wave propagating in a homogeneous medium, the amplitude is given by

$$A(x,t) = A_0 e^{i(kx-\omega t)}, \tag{11}$$

where ω is the angular frequency and k is the wavenumber. Attenuation may be introduced mathematically by allowing either the frequency or wavenumber to be complex. In the latter case,

$$k = k_r + i\alpha \tag{12}$$

so that

$$A(x,t) = A_0 e^{-\alpha x} e^{i(k_r x - \omega t)}, \tag{13}$$

where the phase velocity is

$$v = \frac{\omega}{k_r}. \tag{14}$$

Now I show the derivation of complex velocity v_c for visco-acoustic modeling.

$$\begin{aligned} v_c &= \frac{\omega}{k} \\ &= \frac{\omega}{k_r + i\alpha} \\ &= \frac{\omega}{k_r + i\alpha} \cdot \frac{k_r - i\alpha}{k_r - i\alpha} \\ &= \frac{\omega\,(k_r - i\alpha)}{k_r^2 + \alpha^2} \end{aligned} \tag{15}$$

Rearranging equation 9 yields

$$\alpha = \frac{\pi f}{Qv} = \frac{\omega}{2Qv}. \tag{16}$$

Substituting equation 16 into equation 15, we obtain

$$\begin{aligned} v_c &= \frac{\omega\left(k_r - i\frac{i\omega}{2Qv}\right)}{k_r^2 + \left(\frac{\omega}{2Qv}\right)^2} \\ &= \frac{\omega k_r\left(1 - \frac{i\omega}{2Qvk_r}\right)}{k_r^2\left(1 + \left[\frac{\omega}{2Qvk_r}\right]^2\right)} \end{aligned} \tag{17}$$

Using equation 14, equation 17 can be rewritten as

$$\begin{aligned} v_c &= v\frac{\left(1 - \frac{i}{2Q}\right)}{\left(1 + \frac{1}{(2Q)^2}\right)} \\ &= v\left(1 + \frac{i}{2Q}\right)^{-1} \end{aligned} \tag{18}$$

Equation 18 is then used to combine velocity and quality factor models to simulate visco-acoustic wave propagation which takes into account the effect of anelastic attenuation.