Image Enhancement in the Spatial Domain
Image Histogram

- “Image histogram is a plot of the number of pixels with each possible brightness level.”

- “The visibility of structures can be improved by stretching the contrast so that the value of pixels are reassigned to cover the entire available range.”

\[ P' = 255 \frac{P - \text{darkest}}{\text{brightest}-\text{darkest}} \]

Russ and Neal (2016, p. 245)
Contrast Expansion

The linear mapping was applied to the original image (left) and provided a higher-contrast image (right).

Russ and Neal (2016, p. 246)
Contrast Expansion for Color Images

- Convert RGB to HSI or Lab color space.
- Apply the linear mapping to intensity, luminance, or lightness scale while leaving the color information unchanged.

Original image           expanding only intensity           expanding RGB values

Russ and Neal (2016, p. 248)
The stored value of brightness of each pixel can be mapped to a displayed value using a transfer function.

Red line is the transfer function.

Contrast Manipulation

Russ and Neal (2016, p. 250)
Contrast Manipulation

- Gamma correction uses a nonlinear transfer function

\[
\left( \frac{\text{display}}{255} \right) = \left( \frac{\text{original}}{255} \right)^\gamma
\]

Russ and Neal (2016, p. 252)
Contrast Manipulation

“A transfer function can be created arbitrarily to reveal important details of an image.”

Russ and Neal (2016, p. 252-253)
Histogram Equalization

- Histogram equalization uses the **cumulative curve** (summation of pixel values) as the transfer function.
- The result uses all available brightness values equally.

\[
f(j) = \frac{255}{T} \sum_{i=0}^{j} N_i
\]

*\( T \) is the total number of pixels.
*\( j \) is brightness level.
*\( N_i \) is the number of pixels at brightness level \( i \).

Russ and Neal (2016, p. 253)
Histogram Equalization

Russ and Neal (2016, p. 254)
Histogram Equalization

Russ and Neal (2016, p. 255)
Histogram Equalization for Color Image

- Convert RGB to HSI and apply histogram equalization to intensity only. Leave color information unchanged.

Russ and Neal (2016, p. 257)
Histogram Equalization for Color Image

Russ and Neal (2016, p. 258)
Local Equalization

- Histogram modification of local image regions can improve the visibility of some feature in an image.

Original image

Images after local equalization

Radius = 6

Radius = 3

Russ and Neal (2016, p. 257)
Variance Equalization

- Variance equalization also uses a moving neighborhood filter like local equalization.
- Variance equalization only modifies the central pixel while local equalization modifies all pixels within the filtering region.
- “Statistical variance of the pixels in the region is computed and compared to that of the entire image, and the pixel values are adjusted to match the local variance to the global.”

Russ and Neal (2016, p. 257)
Variance Equalization

(a) original.
(b) local brightness equalization.
(c) blend of (a) and (b)
(d) local variance equalization

Russ and Neal (2016, p. 260)
Variance Equalization for Color Image

Convert RGB to HSI and apply variance equalization to intensity only.

Original

Variance equalization with radius = 6

Russ and Neal (2016, p. 261)
Variance Equalization for Color Image

Median filter (radius = 2) + variance equalization

2/3 of (b) + 1/3 of (c)

Russ and Neal (2016, p. 261)
Laplacian Sharpening

- Laplacian sharpening is a filter that enhances edges.
- The convolution kernel of a 3x3 Laplacian filter is
  \[
  \begin{bmatrix}
  -1 & -1 & -1 \\
  -1 & 8 & -1 \\
  -1 & -1 & -1
  \end{bmatrix}
  \]
  It is an approximation to the Laplacian operator
  \[
  \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
  \]

Russ and Neal (2016, p. 261)
**Laplacian Sharpening**

The convolution kernel of a sharpening operator is

\[
\begin{bmatrix}
-1 & -1 & -1 \\
-1 & 9 & -1 \\
-1 & -1 & -1 \\
\end{bmatrix}
\]

Russ and Neal (2016, p. 263)
Unsharp Mask

(a) Image of M101
(b) Out-of-focus negative of (a)
(c) Combine (a) and (b)
(d) Add (a) and (c) produces the unsharp mask result

Russ and Neal (2016, p. 269)
(a) Original image
(b) Gaussian smoothed
(c) (a) − (b)
(d) (a) + (c)
Unsharp Mask

Russ and Neal (2016, p. 270)
Difference of Gaussians (DoG)

- Subtract one smoothed version of the image from another having a different degree of smoothing.

\[
DoG = \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_1^2}} - \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{x^2}{2\sigma_2^2}}
\]

Russ and Neal (2016, p. 271)
(a) Original image
(b) 3x3 sharpening filter
(c) DoG using $\sigma = 0.5$ and $\sigma = 2.5$ pixels

Russ and Neal (2016, p. 271)
Derivative filters are suitable for images having features with a principal orientation.

(a) Chromatography in which proteins are spread along lanes in an electric field. (b) horizontal derivative using a 1-pixel high kernel. (c) horizontal derivative using a 5-pixel high kernel for noise reduction.
Typical kernels for horizontal first derivative are

\[
\begin{bmatrix}
1 & 0 & -1 \\
1 & -1 \\
2 & 0 & -2 \\
1 & 0 & -1
\end{bmatrix}, \quad
\begin{bmatrix}
2 & 0 & -2 \\
4 & 0 & -4 \\
2 & 0 & -2 \\
1 & 0 & -1
\end{bmatrix}
\]

Kernels for first derivative in tilted direction

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1
\end{bmatrix}, \quad
\begin{bmatrix}
2 & 1 & 0 \\
1 & 0 & -1 \\
0 & -1 & -2
\end{bmatrix}
\]
3x3 derivative

3x3 Laplacian

3x3 sharpening
Derivative
Edges of features are represented as a step in brightness.

“Laplacian filter, based on second derivative, gives a larger response to a line than to a step, and to a point than to a line.”

“Directional first derivative only highlights edges in a direction perpendicular to their orientation.”

One of earliest filters to locate edges of arbitrary orientation is the *Roberts’ cross operator* – applications of two first derivatives of brightness in perpendicular directions:

\[
\begin{bmatrix}
1 & 0 \\
0 & -1 \\
\end{bmatrix}
\begin{bmatrix}
0 & 1 \\
-1 & 0 \\
\end{bmatrix}
\]
Edge Detectors

- A common method to combine two orthogonal vectors is to compute the magnitude of the resulting vector.
- An example is the Sobel gradient operator which is the magnitude of the local gradient of brightness $B$:

$$|\nabla B| = \sqrt{\left(\frac{\partial B}{\partial x}\right)^2 + \left(\frac{\partial B}{\partial y}\right)^2}$$
### Edge Detectors

<table>
<thead>
<tr>
<th>Original</th>
<th>Horizontal derivative</th>
<th>Absolute value of horizontal derivative</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Original" /></td>
<td><img src="image2" alt="Horizontal derivative" /></td>
<td><img src="image3" alt="Absolute value of horizontal derivative" /></td>
</tr>
<tr>
<td><img src="image4" alt="Vertical derivative" /></td>
<td><img src="image5" alt="Absolute value of vertical derivative" /></td>
<td><img src="image6" alt="Sobel operator" /></td>
</tr>
</tbody>
</table>

- **Original**
- **Horizontal derivative**
- **Absolute value of horizontal derivative**
- **Vertical derivative**
- **Absolute value of vertical derivative**
- **Sobel operator**
### Directional Derivative Filters

<table>
<thead>
<tr>
<th>(Robinson)</th>
<th>(Sobel)</th>
<th>(Prewitt)</th>
<th>(Kirsch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \begin{bmatrix} +1 &amp; 0 &amp; -1 \ +1 &amp; 0 &amp; -1 \ +1 &amp; 0 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} +1 &amp; 0 &amp; -1 \ +2 &amp; 0 &amp; -2 \ +1 &amp; 0 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} +1 &amp; -1 &amp; -1 \ +1 &amp; +2 &amp; -1 \ +1 &amp; -1 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} +5 &amp; -3 &amp; -3 \ +5 &amp; 0 &amp; -3 \ +5 &amp; -3 &amp; -3 \end{bmatrix} ]</td>
</tr>
<tr>
<td>[ \begin{bmatrix} +1 &amp; +1 &amp; 0 \ +1 &amp; 0 &amp; -1 \ 0 &amp; -1 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} +2 &amp; +1 &amp; 0 \ +1 &amp; 0 &amp; -1 \ 0 &amp; -1 &amp; -2 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} +1 &amp; +1 &amp; -1 \ +1 &amp; +2 &amp; -1 \ -1 &amp; -1 &amp; -1 \end{bmatrix} ]</td>
<td>[ \begin{bmatrix} +5 &amp; +5 &amp; -3 \ +5 &amp; 0 &amp; -3 \ -3 &amp; -3 &amp; -3 \end{bmatrix} ]</td>
</tr>
</tbody>
</table>
Edge Detectors

Original                      3x3 Sobel filter                     7x7 Sobel filter
The direction of the gradient vector can be computed by

\[
\text{Direction} = \tan^{-1}\left(\frac{\frac{\partial B}{\partial y}}{\frac{\partial B}{\partial x}}\right)
\]
Edge Orientation

SEM image of eggshell membrane

Sobel direction operator

Rose diagram of fiber orientations
# More Edge Detectors

<table>
<thead>
<tr>
<th>Number</th>
<th>Kernel</th>
<th>Number</th>
<th>Kernel</th>
<th>Number</th>
<th>Kernel</th>
</tr>
</thead>
</table>
| 0)     | \[
\begin{bmatrix}
  +1 & +1 & +1 \\
  +1 & +1 & +1 \\
  +1 & +1 & +1 \\
\end{bmatrix}
\] | 3)     | \[
\begin{bmatrix}
  0 & -1 & +\sqrt{2} \\
  +1 & 0 & -1 \\
 -\sqrt{2} & +1 & 0 \\
\end{bmatrix}
\] | 6)     | \[
\begin{bmatrix}
  -1 & 0 & +1 \\
  0 & 0 & 0 \\
 +1 & 0 & -1 \\
\end{bmatrix}
\] |
| 1)     | \[
\begin{bmatrix}
  -1 & -\sqrt{2} & -1 \\
  0 & 0 & 0 \\
 +1 & +\sqrt{2} & +1 \\
\end{bmatrix}
\] | 4)     | \[
\begin{bmatrix}
  +\sqrt{2} & -1 & 0 \\
 -1 & 0 & +1 \\
 0 & +1 & -\sqrt{2} \\
\end{bmatrix}
\] | 7)     | \[
\begin{bmatrix}
  +1 & -2 & +1 \\
 -2 & +4 & -2 \\
 +1 & -2 & +1 \\
\end{bmatrix}
\] |
| 2)     | \[
\begin{bmatrix}
  -2 & 0 & +1 \\
 -\sqrt{2} & 0 & +\sqrt{2} \\
 -1 & 0 & +1 \\
\end{bmatrix}
\] | 5)     | \[
\begin{bmatrix}
  0 & +1 & 0 \\
 -1 & 0 & +1 \\
 0 & -1 & 0 \\
\end{bmatrix}
\] | 8)     | \[
\begin{bmatrix}
  -2 & +1 & -2 \\
 +1 & +4 & +1 \\
 -2 & +1 & -2 \\
\end{bmatrix}
\] |
References