

Magnetic Pendulum

Chaiwoot Boonyasiriwat

September 4, 2015

Goals

- To understand the phenomenon of undamped oscillation in a magnetic pendulum
- To learn how to use the Vernier LABQUEST2 data logger and a rotational sensor
- To learn how to use function generator and signal amplifier

Tasks

- Learn how to use the Vernier LABQUEST2 data logger and export data for graph plotting
- Find frequencies of the sine signal from functional generator that can provide undamped oscillation of the pendulum. You will have to also vary the volume of signal amplifier to increase the AC current passing through the electromagnet (electric coil).
- Explain the phenomenon of damped and undamped oscillations
- Compare the phase space plots of damped and undamped oscillations

Questions

- What is electromagnet?
- Why pendulum oscillations are not the same each time the pendulum was released from the same angular position? Hint: The apparatus in this experiment has no control of the initial phase of the driving torque ϕ_f .
- When will undamped oscillation occur?
- What are the differences between magnetic pendulum and the driven and damped harmonic oscillation?
- Why is the amplitude of the driving torque angle-dependent?

Relevant concepts

- Simple harmonic oscillation for a simple pendulum is governed by the equation

$$\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0$$

where θ is the angular displacement, $\omega_0 = \sqrt{g/l}$ is the natural frequency, g is gravitational acceleration, l is pendulum arm length. The solution of the simple harmonic oscillation equation is the undamped oscillation given by

$$\theta(t) = A \sin(\omega_0 t + \phi_0)$$

where A is the amplitude, ϕ_0 is the initial phase of oscillation.

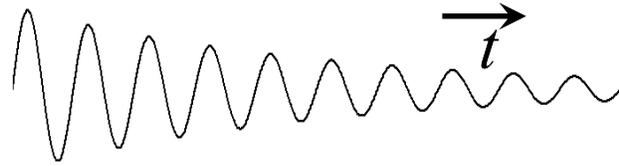
- Damped harmonic oscillation is governed by the equation

$$\frac{d^2\theta}{dt^2} + a \frac{d\theta}{dt} + \omega_0^2 \theta = 0$$

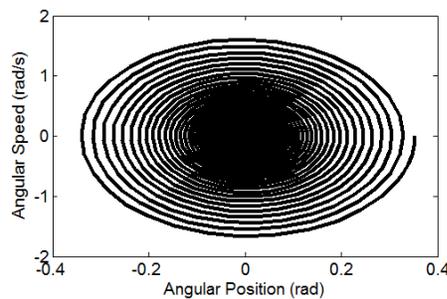
where a is the damping coefficient. The solution of the damped harmonic oscillation equation is given by

$$\theta(t) = Ae^{-\frac{at}{2}} \sin(\omega_a t + \phi_0)$$

where $\omega_a = \sqrt{\omega_0^2 - a^2/4}$ is the initial phase of oscillation. The right figure shows an example of damped harmonic oscillation



A phase space plot (angular displacement versus angular velocity) of a damped harmonic oscillation is a clockwise spiral as shown in below figure. Note that the trajectory eventually reaches the origin.



- Driven and damped harmonic oscillation is governed by the equation

$$\frac{d^2\theta}{dt^2} + a \frac{d\theta}{dt} + \omega_0^2 \theta = F \sin(\omega_f t + \phi_f)$$

where F is the amplitude of the driving torque, ω_f is the driving angular frequency, and ϕ_f is the initial phase of the driving torque. This mathematical model can explain the well-known phenomenon of linear resonance in which maximum oscillation amplitude occurs when the driving frequency is equal to the natural frequency.

- Magnetic pendulum is governed by the equation

$$\frac{d^2\theta}{dt^2} + a \frac{d\theta}{dt} + \omega_0^2 \sin \theta = F(\theta) \sin(\omega_f t + \phi_f)$$

Note that $\sin \theta$ is no longer approximated by θ and $F(\theta)$ is now a function of θ .